

PeV scale Left-Right symmetry and baryon asymmetry of the Universe

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Abstract

We study the cosmology of two versions of supersymmetric Left-Right symmetric model. The scale of the $B-L$ symmetry breaking in these models is naturally low, $10^4 - 10^6$ GeV. Spontaneous breakdown of parity is accompanied by a first order phase transition. We simulate the domain walls of the phase transition and show that they provide requisite conditions, specifically, CP violating phase needed for leptogenesis. Additionally soft resonant leptogenesis is conditionally viable in the two models considered. Some of the parameters in the soft supersymmetry breaking terms are shown to be constrained from these considerations. It is argued that the models may be testable in upcoming collider and cosmology experiments.

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I. INTRODUCTION

Left-Right symmetric model [1, 2, 3, 4, 5] is a simple extension of the Standard Model (SM) [6, 7, 8]. From a theoretical point of view it provides an elegant explanation for the conservation of $B - L$ which automatically becomes a gauge charge, and as a bonus provides a natural explanation for the meaning of the electroweak hypercharge. The new gauge symmetries required constitute the group $SU(2)_R \otimes U(1)_{B-L}$. The model has long been understood as a possible intermediate state in the $SO(10)$ [9, 10] grand unified theory (GUT). However unification in $SO(10)$ generically also forces the possible intermediate scale of Left-Right symmetry to be high and therefore inaccessible to accelerators. On the other hand, the less restrictive principle of exact Left-Right symmetry is still appealing though it leaves the $U(1)_{B-L}$ charge unrelated to the two identical charges of $SU(2)_L$ and $SU(2)_R$. As for the fermion sector the presence of right handed neutrino states in the theory allows the possibility of explaining the smallness of the observed neutrino masses [11, 12, 13, 14] from the see-saw mechanism [15, 16, 17, 18]. While the scale of Majorana masses is no longer as high as in the conventional see-saw expectations, the PeV scale still permits [19] explaining the smallness of the light neutrino mass scale for at least certain textures of fermion mass parameters. It is therefore worth exploring the possibility that the scale of Left-Right symmetry be the PeV scale, potentially testable in colliders.

Whether we follow the GUT proposal or the PeV scale possibility, the large hierarchy between the mass scales $M_{EW} \sim 250\text{GeV}$ of electroweak symmetry and $M_{GUT} \sim 10^{15}\text{GeV}$ is difficult to understand within the Higgs paradigm. While the Higgs sector of the Standard Model is poorly understood, it is nevertheless very successful. We therefore speculate that the breaking of both the $SU(2)_L$ and $SU(2)_R$ being at a comparable scale will have a similar explanation, possibly a comprehensive one including both. There remains the need to understand the hierarchy with respect to a larger mass scale either the GUT scale or the Planck scale. In this paper we assume supersymmetry (SUSY) to be the mechanism to stabilize the hierarchy beyond the electroweak scale [20, 21], in other words we assume TeV scale SUSY¹. We study what has been called the minimal supersymmetric Left-Right symmetric model (MSLRM) [25] with the gauge group $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ augmented

¹ See for instance [22, 23, 24] and references therein.

by parity P exchanging L and R sectors. Lee et al. [26] have studied a similar model with the gauge group $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$ and connected it to cosmological phenomena, specifically inflation. Our discussion differs in being specifically PeV scale.

In the MSLRM class of Left-Right symmetric models, spontaneous gauge symmetry breaking required to recover SM phenomenology also leads to observed parity breaking. However, for cosmological reasons it is not sufficient to ensure local breakdown of parity. We have earlier proposed [27] that the occurrence of the SM like sector globally is connected to the SUSY breaking effects from the hidden sector. Another approach to implementing the global uniformity of parity breaking is to have terms induced by gauge symmetry breaking which signal explicit parity breaking [28, 29]. This model has been dubbed MSLR \overline{P} . In earlier papers we have explored the overall cosmological setting for these models and traced issues such as removal of unwanted relics and a successful completion of the first order phase transition. Here we show that sufficient conditions exist in the model to provide for the leptogenesis required to explain the baryon asymmetry of the Universe.

A possible implementation of this idea follows the thermal leptogenesis [30] route. This however has been shown to generically require the scale of majorana neutrino mass, equivalently, in our model the scale of $B - L$ breaking to be 10^{11} - 10^{13} GeV [31, 32], with a more optimistic constraint $M_{B-L} > 10^9$ GeV [33, 34]. This situation is not improved [35, 36, 37, 38] by assistance from cosmic string induced violation [39, 40, 41] of lepton number [42]. On the other hand, it has been shown [19, 43] that the only real requirement imposed by Leptogenesis is that the presence of heavy neutrinos should not erase lepton asymmetry generated by a given mechanism, possibly non-thermal. This places the modest bound $M_1 > 10^4$ GeV, on the mass of the lightest of the heavy majorana neutrinos. A scenario which exploits this window and relies on supersymmetry is the “soft leptogenesis”, [44, 45, 46, 47] relying on the decay of scalar superpartners of neutrino and a high degree of degeneracy [48] in the mass eigenvalues due to soft SUSY breaking terms.

Another possibility for leptogenesis arises from the fact that generically the Left-Right breaking phase transition is intrinsically a first order phase transition. Due to the presence of lepton number violating processes, the problem of leptogenesis then becomes analogous to that explored for the electroweak phase transition [49], provided a source for CP asymmetry can be found. It has been shown [50] that the domain walls arising during the phase transition generically give spatially varying complex masses to neutrinos. Here we explore

the parameter space required in the two variants of Left-Right symmetric model to ensure the required leptogenesis.

The paper is organized as follows. In the next sections II and III we review the models being considered. In sec. IV we discuss the cosmological evolution characteristic of each of the models, along with the constraints that can be obtained on the soft parameters of the models by the demand that the phase transition is completed successfully. In V we identify the soft parameters in the model that can be constrained by the demand for soft leptogenesis. In sec.s VI and VII we detail the mechanism of leptogenesis by the domain wall (DW) structure of the phase transition and then obtain numerical solutions which support the possibility of this mechanism to operate in the two models. Conclusions are summarized in sec. VIII.

II. MSLRM

The standard Left-Right symmetric model is based on the gauge group $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. The right handed charged leptons which were singlet in standard model (SM), form doublets with respective right handed neutrino species ν_R under $SU(2)_R$ in this model. In the same manner, the right handed up and down quarks of each generation which were singlets in SM, form doublets under $SU(2)_R$. The Higgs sector has two triplets (Δ 's), and a bidoublet (Φ). In minimal supersymmetric Left-Right model (MSLRM) [25], the bidoublet is doubled to have non-vanishing Cabibo-Kobayashi-Maskawa matrix and the triplets are doubled for reasons of anomaly cancellation. The quark and leptonic sectors along with their quantum numbers are represented below.

$$\begin{aligned} Q &= (3, 2, 1, 1/3), & Q_c &= (3^*, 1, 2, -1/3), \\ L &= (1, 2, 1, -1), & L_c &= (1, 1, 2, 1), \end{aligned} \tag{1}$$

where we have suppressed the generation index. The minimal set of Higgs superfields required is,

$$\begin{aligned} \Phi_i &= (1, 2, 2, 0), & i &= 1, 2, \\ \Delta &= (1, 3, 1, 2), & \bar{\Delta} &= (1, 3, 1, -2), \\ \Delta_c &= (1, 1, 3, -2), & \bar{\Delta}_c &= (1, 1, 3, 2). \end{aligned} \tag{2}$$

Under discrete parity symmetry the fields are prescribed to transform as,

$$\begin{aligned} Q &\leftrightarrow Q_c^*, & L &\leftrightarrow L_c^*, & \Phi_i &\leftrightarrow \Phi_i^\dagger, \\ \Delta &\leftrightarrow \Delta_c^*, & \bar{\Delta} &\leftrightarrow \bar{\Delta}_c^*. \end{aligned} \quad (3)$$

However, this minimal model is unable to break parity spontaneously [51, 52]. A parity odd singlet solves this problem [53], but this also breaks electromagnetic charge invariance [51]. Breaking R parity and introducing non-renormalizable terms solves this problem. A more appealing way out is to introduce a pair of scalar triplets (Ω, Ω_c) , which are even under parity viz., $\Omega \leftrightarrow \Omega_c^*$ [27, 29, 54]. The quantum numbers for the two fields are,

$$\Omega = (1, 3, 1, 0), \quad \Omega_c = (1, 1, 3, 0). \quad (4)$$

The superpotential for this model was given in [54]. It is almost the same as the superpotential given later in this paper, in sec. III, eq. (11) from which it can be obtained with Ω_c replaced by $-\Omega_c$. Since in this class of models, we consider supersymmetry to be broken only at the electroweak scale, we can safely employ the F -flatness and D -flatness conditions to obtain the vacua of the theory. The F and D flat conditions for MSLRM are given in ref [54] and again are similar in nature to the one we have worked out in appendix A for the modified version of this model discussed in sec. III. These F and D flat conditions imply the existence of the following set of vacuum expectation values (vev's) for the Higgs fields as one of the possibilities.

$$\begin{aligned} \langle \Omega \rangle &= 0, & \langle \Delta \rangle &= 0, & \langle \bar{\Delta} \rangle &= 0, \\ \langle \Omega_c \rangle &= \begin{pmatrix} \omega_c & 0 \\ 0 & -\omega_c \end{pmatrix}, & \langle \Delta_c \rangle &= \begin{pmatrix} 0 & 0 \\ d_c & 0 \end{pmatrix}, & \langle \bar{\Delta}_c \rangle &= \begin{pmatrix} 0 & \bar{d}_c \\ 0 & 0 \end{pmatrix}. \end{aligned} \quad (5)$$

The stages of breaking required to implement parity breaking and avoid electromagnetic charge breaking vacua, are as follows: first the Ω 's get a vev at a scale M_R , which breaks $SU(2)_R$ to its subgroup $U(1)_R$, but conserving $B - L$ charge. At a lower scale M_{B-L} , the triplets get vev to break $U(1)_R \otimes U(1)_{B-L}$ to $U(1)_Y$. Thus, at low scale MSLRM breaks exactly to minimal supersymmetric standard model (MSSM).

From the F and D flatness conditions we are led to the following solution for the vev's

[27, 29, 54]

$$\begin{aligned} |\omega| &= \left| \frac{m_\Delta}{a} \right| \equiv M_R, \\ |d| = |\bar{d}| &= \left| \frac{2m_\Delta m_\Omega}{a^2} \right|^{1/2} \equiv M_{B-L} \end{aligned} \quad (6)$$

For parity breakdown we must have $M_R \gg M_{B-L}$, which is accomplished if we have $m_\Delta \gg m_\Omega$. If the mass scale m_Ω originates from the soft terms, then we can accept the approach of Ref [55] that $m_\Omega \simeq M_{EW}$. This in turn would mean that m_Ω is of the same order as the gravitino mass $m_{3/2}$. This leads us to the relation

$$M_{B-L}^2 \simeq M_R M_{EW}. \quad (7)$$

Thus, we have only one effective new mass scale, either M_R or M_{B-L} . Now if we consider $M_{B-L} \sim 10^4$ GeV, then $M_R \sim 10^6$ GeV. On the other hand, $M_{B-L} \sim 10^6$ GeV, if we choose M_R to have the largest possible value $\sim \sqrt{M_{Pl} M_{EW}} \sim 10^{10}$ GeV, beyond which non-renormalizable terms will be relevant. Thus the model is workable in a wide range of values, but the lower range values make the model verifiable in the colliders.

The above solution for the vev's, is not unique. Due to Left-Right symmetric nature of the original theory, an alternative set of vev's permitted by the F and D flatness conditions are,

$$\begin{aligned} \langle \Omega \rangle &= \begin{pmatrix} \omega & 0 \\ 0 & -\omega \end{pmatrix}, & \langle \Delta \rangle &= \begin{pmatrix} 0 & 0 \\ d & 0 \end{pmatrix}, & \langle \bar{\Delta} \rangle &= \begin{pmatrix} 0 & \bar{d} \\ 0 & 0 \end{pmatrix}, \\ \langle \Omega_c \rangle &= 0, & \langle \Delta_c \rangle &= 0, & \langle \bar{\Delta}_c \rangle &= 0. \end{aligned} \quad (8)$$

Due to the possibility of alternative set of Higgs vacua, in the early universe, parity breakdown does not select unique ground state and formation of domain walls (DW) is inevitable. As this contradicts present observable cosmology the model must have an inbuilt asymmetry to remove the domain walls. Since the superpotential doesn't allow such asymmetry in the present model, we depend on the soft terms to do the job.

The mechanism which induces the soft terms can arise due to gravitational effects in the gravity mediated supersymmetry breaking. In gauge mediated supersymmetry breaking (GMSB), the soft terms can arise due to the messenger sector, the hidden sector or both. In the next section III however, we look for an alternative possibility for the breaking parity, which arises naturally out of the Higgs sector.

III. MSLR \mathcal{P}

In this section we consider another possibility for parity breaking which takes place within the Higgs sector. The idea was first considered by Chang *et al.* [28], for the non-susy model $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes P$ where P denotes parity symmetry. To break parity an extra Higgs singlet η which is odd under P parity was introduced .i.e $\eta \leftrightarrow -\eta$. As such the potential of the model has a term of the form

$$V_{\eta\Delta} \sim M\eta(\Delta_L^\dagger \Delta_L - \Delta_R^\dagger \Delta_R), \quad (9)$$

where the notation is self-evident. Thus, when at a high scale M_P , the singlet η gets a vev, the effective masses of the left and right triplet Higgs masses become different, thus explicitly breaking P parity, without affecting $SU(2)_R$. However, in SUSY, a parity odd singlet in the theory would generate the problems of charge breaking vacua as discussed by Kuchimanchi and Mohapatra [51]. To avoid this, but to implement the idea of Chang *et al.* we propose an alternative SUSY model based on the group $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes P$ with a pair triplets (Ω, Ω_c) which are odd under parity. This model was discussed in an earlier paper [29] and was named MSLR \mathcal{P} . Under parity,

$$\begin{aligned} Q &\leftrightarrow Q_c^*, & L &\leftrightarrow L_c^*, & \Phi_i &\leftrightarrow \Phi_i^\dagger, \\ \Delta &\leftrightarrow \Delta_c^*, & \bar{\Delta} &\leftrightarrow \bar{\Delta}_c^*, & \Omega &\leftrightarrow -\Omega_c^*. \end{aligned} \quad (10)$$

The superpotential for this parity symmetry becomes,

$$\begin{aligned} W_{LR} = & \mathbf{h}_l^{(i)} L^T \tau_2 \Phi_i \tau_2 L_c + \mathbf{h}_q^{(i)} Q^T \tau_2 \Phi_i \tau_2 Q_c + i\mathbf{f} L^T \tau_2 \Delta L + i\mathbf{f} L^{cT} \tau_2 \Delta_c L_c \\ & + m_\Delta \text{Tr} \Delta \bar{\Delta} + m_\Delta \text{Tr} \Delta_c \bar{\Delta}_c + \frac{m_\Omega}{2} \text{Tr} \Omega^2 + \frac{m_\Omega}{2} \text{Tr} \Omega_c^2 \\ & + \mu_{ij} \text{Tr} \tau_2 \Phi_i^T \tau_2 \Phi_j + a \text{Tr} \Delta \Omega \bar{\Delta} - a \text{Tr} \Delta_c \Omega_c \bar{\Delta}_c \\ & + \alpha_{ij} \text{Tr} \Omega \Phi_i \tau_2 \Phi_j^T \tau_2 - \alpha_{ij} \text{Tr} \Omega_c \Phi_i^T \tau_2 \Phi_j \tau_2, \end{aligned} \quad (11)$$

where color and flavor indices have been suppressed. Further, $\mathbf{h}_q^{(i)} = \mathbf{h}_q^{(i)\dagger}$, $\mathbf{h}_l^{(i)} = \mathbf{h}_l^{(i)\dagger}$, $\mu_{ij} = \mu_{ji} = \mu_{ij}^*$, $\alpha_{ij} = -\alpha_{ji}$. Finally, \mathbf{f} , \mathbf{h} are real symmetric matrices with respect to flavor indices.

The F and D flatness conditions derived from this superpotential are presented in appendix A. However, the effective potential for the scalar fields which is determined from modulus square of the D terms remains the same as for the MSLRM at least for the form of

the ansatz of the vev's we have chosen. As such the resulting solution for the vev's remains identical to eq. (6). The difference in the effective potential shows up in the soft terms as will be shown later. Due to soft terms, below the scale M_R the effective mass contributions to Δ and $\bar{\Delta}$ become larger than those of Δ_c and $\bar{\Delta}_c$. The cosmological consequence of this is manifested after the M_{B-L} phase transition when the Δ 's become massive. Unlike MSLRM where the DW are destabilized only after the soft terms become significant, i.e., at the electroweak scale, the DW in this case become unstable immediately after M_{B-L} . Leptogenesis therefore commences immediately below this scale and the scenario becomes qualitatively different from that for the MSLRM.

In the next section we elaborate in detail the areas where the two models MSLRM and MSLRP differ from the cosmological point of view.

IV. COSMOLOGY OF BREAKING

In this section we recapitulate the cosmology of these models. In the two models MSLRM and MSLRP the stages of breaking are slightly different as shown in Table (I). Domain walls form in both the models at the scale M_R , when the Ω fields get vev. These DW come to dominate the evolution of the Universe and is responsible for the onset of a secondary inflation. This secondary inflation removes gravitinos and other relic abundances which were regenerated during the reheating stage after the primordial inflation ended [27, 29]. At the scale M_{B-L} , the triplet Δ 's get vev. At this epoch the effective mass of the left-handed Δ 's is essentially different than those of right-handed Δ 's in MSLRP. As such at this stage DW are destabilized and leptogenesis begins in MSLRP unlike in MSLRM. SUSY breaking is mediated from the hidden sector to the visible sector in both the models at the scale M_S . The soft terms which become relevant at this scale break the parity in MSLRM. Thus the DW become destabilized in MSLRM at M_S , thus beginning the process of leptogenesis. The walls finally disappear in MSLRP at a scale $T_D \sim 10 - 10^3$ GeV and in MSLRM at $T_D \sim 10 - 10^2$ GeV. Subsequently standard cosmology takes over after this.

A handle on the explicit symmetry breaking parameters of the two models can be obtained by noting that there should exist sufficient wall tension for the walls to disappear before a desirable temperature scale T_D . It has been observed in [56] that the free energy density difference $\delta\rho$ between the vacua, which determines the pressure difference across a domain

Cosmology	Scale	Symmetry Group	MSLR \mathcal{P} (GeV)	MSLRM (GeV)
Ω or Ω_c get vev.		$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$		
Onset of wall dominated secondary inflation.	M_R	\downarrow	10^6	10^6
Higgs triplet ($\Delta's$) get vev	M_{B-L}	\downarrow	10^4	10^4
End of inflation and beginning of L-genesis	M_{B-L}		10^4	—
	M_S		—	10^3
SUSY breaking	M_S	$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ (SUSY) \downarrow	10^3	10^3
Wall disappearance temperature	T_D		$10 - 10^3$	$10 - 10^2$
Secondary reheat temperature	T_R^s		$10^3 - 10^4$	10^3
Electroweak breaking	M_{EW}	$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ (non-SUSY) \downarrow	10^2	10^2
Standard Model		$SU(3)_c \otimes U(1)_{EW}$		

TABLE I: Pattern of symmetry breaking and the slightly different sequence of associated cosmological events in the two classes of models

wall should be of the order

$$\delta\rho \sim T_D^4 \quad (12)$$

in order for the DW structure to disappear at the scale T_D .

$T_D/\text{GeV} \sim$	10^{-1}	1	10	10^2	10^3
$(m^2 - m'^2)/\text{GeV}^2 \sim$	10^{-12}	10^{-8}	10^{-4}	1	10^4
$(\beta_1 - \beta_2)/\text{GeV}^2 \sim$	10^{-16}	10^{-12}	10^{-8}	10^{-4}	1

TABLE II: Differences in values of soft supersymmetry breaking parameters of MSLRM, for a range of domain wall decay temperature values T_D . The differences signify the extent of parity breaking.

A. Consistent cosmology : MSLRM

The possible source for breaking the parity symmetry of the MSLRM lies in soft terms with the assumption that the hidden sector, or in case of GMSB also perhaps the messenger sector does not obey the parity of the visible sector model. For gravity mediated breaking this can be achieved in a natural way since a discrete symmetry can be generically broken by gravity effects. We present the possible soft terms for MSLRM below.

$$\begin{aligned}
\mathcal{L}_{soft} = & \alpha_1 \text{Tr}(\Delta \Omega \Delta^\dagger) + \alpha_2 \text{Tr}(\bar{\Delta} \Omega \bar{\Delta}^\dagger) + \alpha_3 \text{Tr}(\Delta_c \Omega_c \Delta_c^\dagger) + \alpha_4 \text{Tr}(\bar{\Delta}_c \Omega_c \bar{\Delta}_c^\dagger) \\
& + m_1^2 \text{Tr}(\Delta \Delta^\dagger) + m_2^2 \text{Tr}(\bar{\Delta} \bar{\Delta}^\dagger) + m_3^2 \text{Tr}(\Delta_c \Delta_c^\dagger) + m_4^2 \text{Tr}(\bar{\Delta}_c \bar{\Delta}_c^\dagger) \\
& + \beta_1 \text{Tr}(\Omega \Omega^\dagger) + \beta_2 \text{Tr}(\Omega_c \Omega_c^\dagger) .
\end{aligned} \tag{13}$$

We can determine the differences between the relevant soft parameters for a range of permissible values of T_D .

In Table II we have taken $d \sim 10^4$ GeV, $\omega \sim 10^6$ GeV and T_D in the range 100 MeV – 10 GeV [57]. The above differences between the values in the left and right sectors is a lower bound on the soft parameters and is very small. Larger values would be acceptable to low energy phenomenology. However if we wish to retain the connection to the hidden sector, and have the advantage of secondary inflation we would want the differences to be close to this bound. As pointed out in [56, 58] an asymmetry $\sim 10^{-12}$ is sufficient to lift the degeneracy between the two sectors.

B. Consistent cosmology : MSLRP

In this model parity breaking is achieved spontaneously within the observable sector below the scale M_R at which the Ω fields acquire vev's. However the breaking is not manifested in the vacuum till the scale M_{B-L} where the Δ fields acquire vev's. For simplicity we assume that the hidden sector responsible for SUSY breaking does not contribute parity breaking terms. This is reasonable since even if the hidden sector breaks this parity the corresponding effects are suppressed by the higher scale of breaking and in the visible sector the parity breaking effects are dominated by the explicit mechanism proposed. Thus at a scale above M_R but at which SUSY is broken in the hidden sector we get induced soft terms respecting this parity. Accordingly, for the Higgs sector the parameters can be chosen such that

$$\begin{aligned}\mathcal{L}_{soft} = & \alpha_1 \text{Tr}(\Delta \Omega \Delta^\dagger) - \alpha_2 \text{Tr}(\bar{\Delta} \Omega \bar{\Delta}^\dagger) - \alpha_1 \text{Tr}(\Delta_c \Omega_c \Delta_c^\dagger) + \alpha_2 \text{Tr}(\bar{\Delta}_c \Omega_c \bar{\Delta}_c^\dagger) \\ & + m_1^2 \text{Tr}(\Delta \Delta^\dagger) + m_2^2 \text{Tr}(\bar{\Delta} \bar{\Delta}^\dagger) + m_1^2 \text{Tr}(\Delta_c \Delta_c^\dagger) + m_2^2 \text{Tr}(\bar{\Delta}_c \bar{\Delta}_c^\dagger) \\ & + \beta \text{Tr}(\Omega \Omega^\dagger) + \beta \text{Tr}(\Omega_c \Omega_c^\dagger) .\end{aligned}\tag{14}$$

These terms remain unimportant at first due to the key assumption leading to MSSM as the effective low energy theory. The SUSY breaking effects become significant only at the electroweak scale. However, below the scale M_R , Ω and Ω_c acquire vev's given by eq. (5) or (8). Further, below the scale M_{B-L} the Δ fields acquire vev's and become massive. The combined contribution from the superpotential and the soft terms to the Δ masses now explicitly encodes the parity breaking,

$$\begin{aligned}\mu_\Delta^2 &= M_\Delta^2 + \alpha_1 \omega, & \mu_{\Delta_c}^2 &= M_\Delta^2 - \alpha_1 \omega, \\ \mu_{\bar{\Delta}}^2 &= M_\Delta^2 + \alpha_2 \omega, & \mu_{\bar{\Delta}_c}^2 &= M_\Delta^2 - \alpha_2 \omega.\end{aligned}\tag{15}$$

where M_Δ^2 is the common contribution from the superpotential. The difference in free energy across the domain wall is now dominated by the differential contribution to the Δ masses

$$\delta\rho_\alpha \equiv 2(\alpha_1 + \alpha_2)\omega d^2,\tag{16}$$

where we have considered $\omega_c \sim \omega$, $d \sim \bar{d} \sim d_c \sim \bar{d}_c$. Now using eq (12) for a range of temperatures ($T_D \sim 10^2 \text{ GeV} - 10^4 \text{ GeV}$), determines the corresponding range of values of coupling constants as

$$(\alpha_1 + \alpha_2) \sim 10^{-6} - 10^2 \text{ GeV},\tag{17}$$

where we have considered $|\omega| \simeq M_R$, $|d| \simeq M_{B-L}$.

V. SUPERSYMMETRY AND LEPTOGENESIS

The supersymmetric Left-Right symmetric models considered here do not favor generic thermal leptogenesis from decay of heavy majorana neutrinos for an intriguing reason. $B-L$ asymmetry in the form of fermion chemical potential is guaranteed to remain zero in the model until the gauged $B-L$ symmetry breaks spontaneously. As can be seen, a generic consequence of symmetry breaking in both the models is a relation among the various mass scales $M_{B-L}^2 \simeq M_{EW} M_R$. Thermal Leptogenesis requires M_{B-L} to be larger than 10^{11} - 10^{13} GeV, which pushes M_R into the Planck scale in light of the above formula. A more optimistic constraint $M_{B-L} > 10^9 \text{ GeV}$ [33, 34] requires Left-Right symmetry to be essentially Grand Unified theory.

However, supersymmetry provides new channels for thermal leptogenesis via out of equilibrium decay of scalar superpartners of leptons [44, 45, 46]. Leptogenesis from scalar sector is free of strong constraints on the Yukawa couplings as happens in thermal leptogenesis from fermion decay [32]. In the mechanism to be discussed, the sneutrino splits into two distinct mass eigenstates due to soft supersymmetry breaking terms. The relevant terms in the superpotential are

$$W_{leptonic} = \mathbf{h}_l^{(i)} L^T \tau_2 \Phi_i \tau_2 L_c + i \mathbf{f} L^T \tau_2 \Delta L + i \mathbf{f} L^{cT} \tau_2 \Delta_c L_c \quad (18)$$

The relevant soft terms (V_{ls}) in our model are given by

$$V_{ls} = A h^{(i)} \tilde{L}^T \tau_2 \Phi_i \tau_2 \tilde{L}_c + i B f \tilde{L}^T \tau_2 \Delta \tilde{L} + i B' f \tilde{L}_c^T \tau_2 \Delta_c \tilde{L}_c + \tilde{m}^2 \tilde{L}^\dagger \tilde{L} + \tilde{m}^2 \tilde{L}_c^\dagger \tilde{L}_c \quad (19)$$

Mixing between the two states of sneutrino generates the CP violation.

Consider the generic model introduced by [45], where the superpotential is given by

$$W = h L H N + \frac{1}{2} M N N, \quad (20)$$

where, L , H and N are the left-handed lepton doublet, the Higgs and the right handed neutrino respectively. Here we have omitted the generation index for simplicity of notation. The SUSY soft breaking terms are given by,

$$V_{\text{soft}} = \left[A h \tilde{L} H \tilde{N} + \frac{1}{2} B M \tilde{N} \tilde{N} + \text{h.c} \right] + \tilde{m}^2 \tilde{N}^\dagger \tilde{N} \quad (21)$$

The mixing between the two eigenstates in the decay of the right-handed sneutrino (\tilde{N}) produces the required CP violation (ϵ). The two eigenstates \tilde{N}_1 and \tilde{N}_2 of the sneutrino,

$\tilde{N} = (\tilde{N}_1 + i\tilde{N}_2)/\sqrt{2}$ are given by

$$M_{\tilde{N}_{1,2}}^2 = M^2 + \tilde{m}^2 \pm BM \quad (22)$$

Due to the near degeneracy of these masses the CP asymmetry can be large. The mechanism has been studied in detail in [59] where it is shown that the constraint on the soft parameter B is

$$B \sim \Gamma \sim 0.1 \text{ eV} \left(\frac{m_\nu}{0.05 \text{ eV}} \right) \left(\frac{M}{\text{TeV}} \right) \quad (23)$$

This is the same as the B parameter in our model introduced in eq. (19). In [59] it is shown that this constraint can be corroborated by collider experiments involving Z' decays. The Z' sector of the model we are considering is similar and similar collider constraints are applicable.

Further, we see that the B required is $O(10^{-12})$ relative to the electroweak scale. This smallness of the value is possible in certain scenarios [60] and is expected in models of hidden sector supersymmetry breaking. Here we see a correspondence between the smallness of this parameter and the parameters in the Higgs sector as determined from the cosmological constraint of disappearance of the DW summarized in sec. IV A. This is a strong indication that we may be able to test the validity of MSLRM by ascertaining its hidden sector breaking scheme and correlating the two cosmological requirements determined from smallness of otherwise unrelated parameters arising from the same mechanism.

VI. LEPTOGENESIS FROM FIRST ORDER PHASE TRANSITION

In addition to the resonant leptogenesis considered in previous section, the models considered here also include natural possibility of non-thermal leptogenesis. The spontaneous breaking of a discrete symmetry automatically makes the Left-Right symmetry breaking phase transition a first order phase transition. The idea is similar to electroweak baryogenesis proposals [49, 61] where there are spontaneously formed bubbles which expand to complete the phase transition, a mechanism also considered in the case of Left-Right symmetric model in [62]. The dynamics of Left-Right breaking phase transition considered here takes into account that due to parity symmetry of the theory both Right-like (unbroken $SU(2)_R$), and Left-like (unbroken $SU(2)_L$) domains are liable to occur at the phase transition. In the models considered here parity is unbroken at the first stage of the symmetry

breaking. The phase transition is accompanied by the spontaneous formation of domain walls separating Left-like and Right-like regions. At a lower scale when parity breaking is signalled, the walls sweep through the Universe ensuring global choice of a unique phase everywhere. The domain walls move irreversibly during this epoch, thereby eliminating the energetically unfavorable phase and providing time irreversibility.

Consider the interaction of neutrinos with the L-R wall, which is encroaching on the energetically disfavored phase. The left-handed neutrinos, ν_L , are massive in this domain, whereas they are massless in the phase behind the wall. More precisely, as per see-saw mechanism, ν_L constitute the principal component of the heavy mass eigenstate in front of the wall but become principal component of the light eigenstate behind the wall, and it is the ν_L whose fate we keep track of. To get leptogenesis, one needs an asymmetry in the reflection and transmission coefficients from the wall between ν_L and its CP conjugate (ν_L^c). This can happen if a CP-violating condensate exists in the wall. This comes from the Dirac mass terms as discussed in [63, 64, 65, 66, 67]. Then there will be a preference for transmission of, say, ν_L . The corresponding excess of antineutrinos (ν_L^c) reflected in front of the wall will quickly equilibrate with ν_L due to helicity-flipping scatterings, whose amplitude is proportional to the large Majorana mass. However the transmitted excess of ν_L survives because it is not coupled to its CP conjugate in the region behind the wall, where the majorana mass contribution from $\langle\Delta\rangle$ and $\langle\Delta_c\rangle$ vanishes.

A quantitative analysis of this effect can be made either in the framework of quantum mechanical reflection, valid for domain walls which are narrow compared to the particles' thermal de Broglie wavelengths, or using the classical force method [63, 64, 65, 66, 67] which gives the dominant contribution for walls with larger widths. We adopt the latter here. The thickness of the wall depends on the shape of the effective quartic potential and we shall here treat the case of thick walls. Further, we assume that the potential energy difference between the two kinds of vacua is small, for example suppressed by Planck scale effects. In this case the pressure difference across the phase boundary is expected to be small, leading to slowly moving walls. The classical CP-violating force of the condensate on a fermion (in our case a neutrino) with momentum component p_x perpendicular to the wall can be shown to be

$$F = \pm \text{sign}(p_x) \frac{1}{2E^2} (m_\nu^2(x) \chi'(x))'. \quad (24)$$

The sign depends on whether the particle is ν_L or ν_L^c , $m_\nu^2(x)$ is the position-dependent mass,

E the energy and χ is the spatially varying CP-violating phase. One can then derive a diffusion equation for the chemical potential μ_L of the ν_L as seen in the wall rest frame:

$$-D_\nu \mu_L'' - v_w \mu_L' + \theta(x) \Gamma_{\text{hf}} \mu_L = S(x). \quad (25)$$

Here D_ν is the neutrino diffusion coefficient, v_w is the velocity of the wall, taken to be moving in the $+x$ direction, Γ_{hf} is the rate of helicity flipping interactions taking place in front of the wall (hence the step function $\theta(x)$), and S is the source term, given by

$$S(x) = -\frac{v_w D_\nu}{\langle \vec{v}^2 \rangle} \langle v_x F(x) \rangle', \quad (26)$$

where \vec{v} is the neutrino velocity and the angular brackets indicate thermal averages. The net lepton number excess can then be calculated from the chemical potential resulting as the solution of eq. (25).

In order to use this formalism it is necessary to establish the presence of a position-dependent phase χ . This is what we turn to in the following discussion of the nature of domain walls in the L-R model.

VII. WALL PROFILES AND CP VIOLATING CONDENSATE

In order for nontrivial effects to be mediated by the walls, the fermion species of interest should get a space-dependent mass from the wall. Furthermore, the CP-violating phase χ should also possess a nonvanishing gradient in the wall interior. We study the minimization of the total energy functional of the scalar sector with this in mind.

The vev's introduced in eq. (5) are in general complex. Some of them can be rendered real by global $SU(2)$ transformations [5, 68]

$$U_L = \begin{pmatrix} e^{i\gamma_L} & 0 \\ 0 & e^{-i\gamma_L} \end{pmatrix}, \quad U_R = \begin{pmatrix} e^{i\gamma_R} & 0 \\ 0 & e^{-i\gamma_R} \end{pmatrix} \quad (27)$$

according to

$$\Phi_1 \rightarrow U_L \Phi_1 U_R^\dagger, \quad \Phi_2 \rightarrow U_L \Phi_2 U_R^\dagger, \quad (28)$$

$$\Delta \rightarrow U_L \Delta U_L^\dagger, \quad \bar{\Delta} \rightarrow U_L \bar{\Delta} U_L^\dagger, \quad (29)$$

$$\Delta_c \rightarrow U_R \Delta_c U_R^\dagger, \quad \bar{\Delta}_c \rightarrow U_R \bar{\Delta}_c U_R^\dagger, \quad (30)$$

$$\Omega \rightarrow U_L \Omega U_L^\dagger, \quad \Omega_c \rightarrow U_R \Omega_c U_R^\dagger. \quad (31)$$

The vev's of the triplets Ω and Ω_c being diagonal are not affected by these transformations. Their phases if any do not enter fermion or sfermion masses. We choose their phases to be real. This leaves us with 16 degrees of freedom in the Higgs sector. These can be parameterized by allowing three of the vev's in the four Δ fields and three of the vev's in the two bidoublets Φ to be complex. Here we present a simpler model. As shown in eq.s (32) and (33), only two of the vev's are chosen to be complex, viz., the Δ and upper component of Φ_1 . The parameters α_{ij} reduce to a single value α times the anti-symmetric matrix ϵ_{ij} , and all the four values of μ_{ij} are chosen to be the same value μ . We have also studied the model with all the allowable phases to be non-zero and find that it does not result in any substantial improvement to the required condition for leptogenesis. The simpler model contains the minimal features to reproduce all the essential features required for leptogenesis.

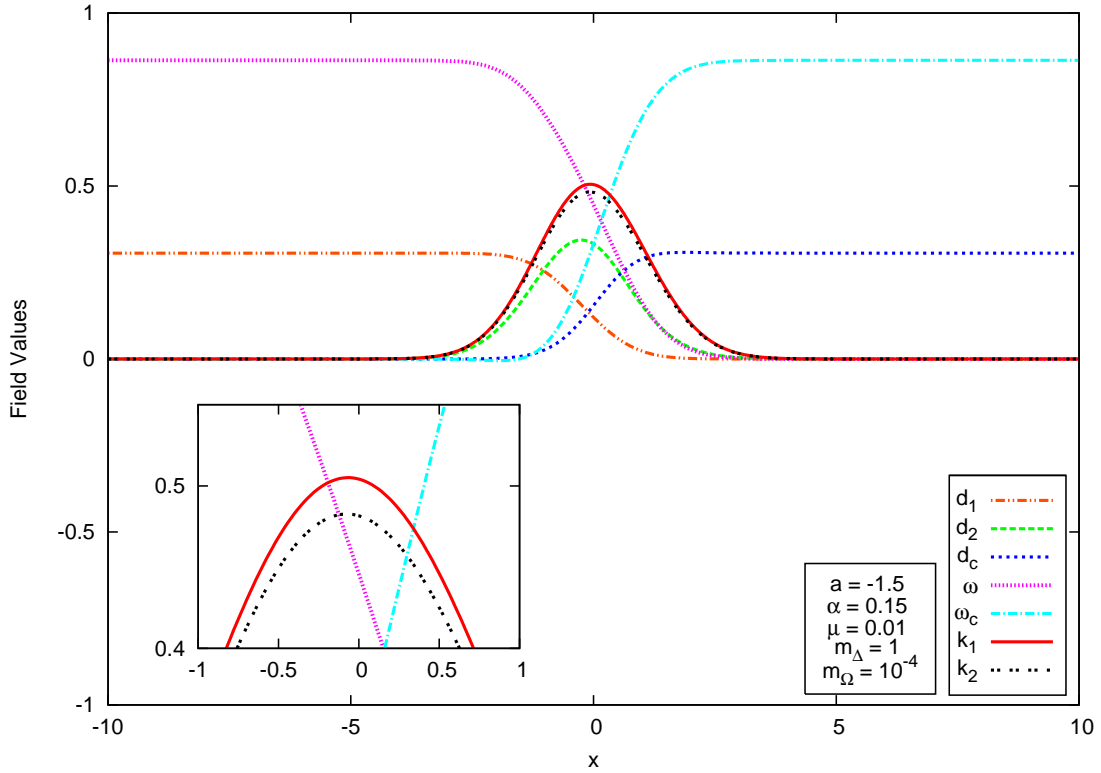


FIG. 1: Domain wall with CP violating condensate in MSLRM. The inset magnifies the behaviour of the k_1 and k_2 near their maximum value.

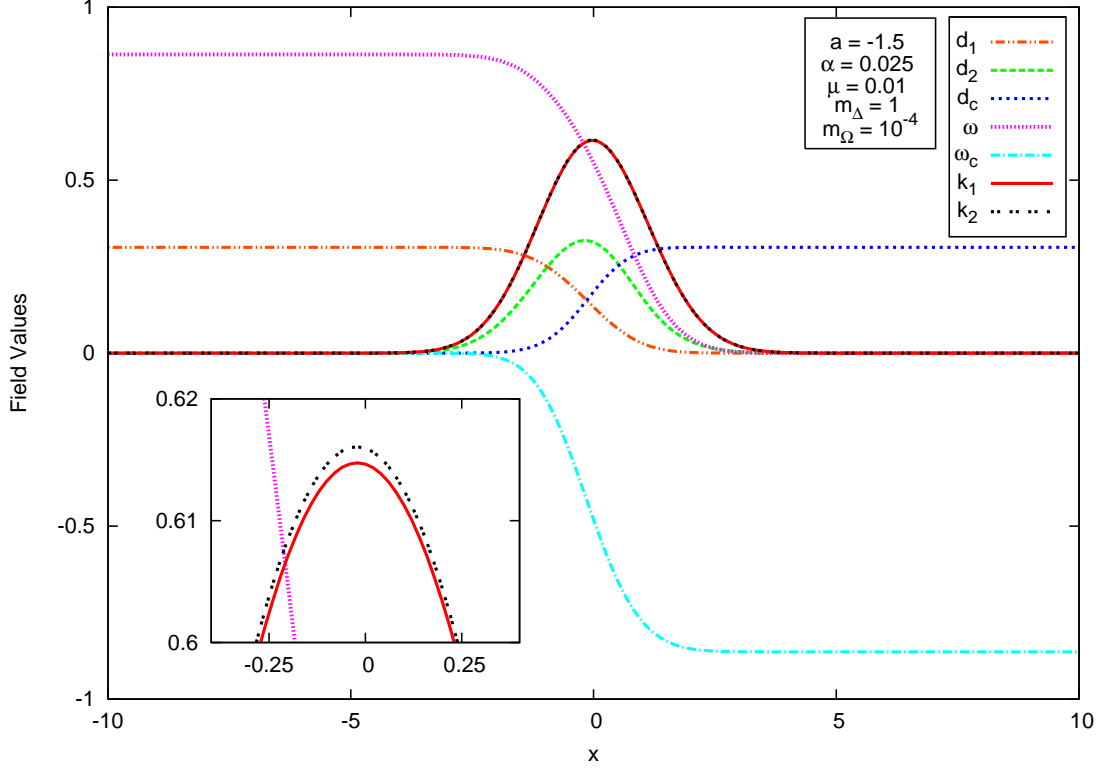


FIG. 2: Domain wall with CP violating condensate in MSLRP. The inset magnifies the behaviour of the k_1 and k_2 near their maximum value.

$$\langle \Omega \rangle = \begin{pmatrix} \omega & 0 \\ 0 & -\omega \end{pmatrix}, \quad \langle \Delta \rangle = \begin{pmatrix} 0 & 0 \\ d_1 + id_2 & 0 \end{pmatrix}, \quad \langle \bar{\Delta} \rangle = \begin{pmatrix} 0 & \sqrt{d_1^2 + d_2^2} \\ 0 & 0 \end{pmatrix}, \quad (32)$$

$$\langle \Omega_c \rangle = \begin{pmatrix} \omega_c & 0 \\ 0 & -\omega_c \end{pmatrix}, \quad \langle \Delta_c \rangle = \begin{pmatrix} 0 & 0 \\ d_c & 0 \end{pmatrix}, \quad \langle \bar{\Delta}_c \rangle = \begin{pmatrix} 0 & d_c \\ 0 & 0 \end{pmatrix}.$$

$$\langle \Phi_1 \rangle = \begin{pmatrix} k_1 + ik_2 & 0 \\ 0 & \sqrt{k_1^2 + k_2^2} \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} \sqrt{k_1^2 + k_2^2} & 0 \\ 0 & \sqrt{k_1^2 + k_2^2} \end{pmatrix} \quad (33)$$

The effective potential obtained by substituting these vev's is given in eq. (B1) in the appendix B. In accordance with the discussion accompanying eq.s (6) and (7), we choose the scale of $M_{B-L} \sim 10^4$ GeV which relates to M_R being of the order of 10^6 GeV.

For numerical simulation the mass parameters are scaled by the largest scale $M_R \sim 10^6$ GeV, i.e. in our simulation $M_R \sim 1$, and other parameters are chosen $m_\Delta \sim \mathcal{O}(1)$ and

α	k_1	k_2	χ	$\chi - \pi/4$
0.001	0.6170	0.6176	0.7859	0.0005
0.005	0.6173	0.6183	0.7861	0.0007
0.01	0.6173	0.6185	0.7863	0.0009
0.025	0.6147	0.6160	0.7864	0.0010
0.035	0.6108	0.6116	0.7860	0.0006
0.045	0.6055	0.6053	0.7852	-0.0001
0.05	0.6023	0.6015	0.7847	-0.0006
0.10	0.5572	0.5467	0.7758	-0.0095
0.15	0.5042	0.4815	0.7624	-0.0229
0.20	0.4564	0.4225	0.7468	-0.0385
0.25	0.4178	0.3740	0.7301	-0.0552
0.30	0.3879	0.3354	0.7128	-0.0725
0.50	0.3233	0.2400	0.6386	-0.1467
0.75	0.2889	0.1745	0.5433	-0.2420
1.00	0.2661	0.1311	0.4579	-0.3274

TABLE III: Peak phase values $\chi = \tan^{-1}(k_2/k_1)$ in both MSLRM and MSLRP for various values of α

$m_\Omega \sim \mathcal{O}(10^{-4})$ as per eq. (6). Parameter μ entering the bidoublet mass terms should be 10^{-4} , however at the scale in question, due to temperature corrections it is expected to be of the same order as M_{B-L} and is chosen 0.01. Eq. (6) dictates that the parameter a be negative and order unity. It is chosen to be -1.5 throughout. The asymptotic values of the fields are such as to minimize the potential under translation invariance. The profiles are then found by relaxation methods. Two examples of the numerically determined profiles are shown in figures 1 and 2.

Electroweak symmetry is unbroken at the epoch under consideration and hence the asymptotic values for k_1 and k_2 are zero. Since both k_1 and k_2 approach the same values asymptotically, the effective asymptotic value of χ is $\pi/4$. The departure from this

value at the maxima of the graphs are listed in table III. It was observed that the difference in k_1 and k_2 profiles, the source of spatially varying CP violating phase χ arises from the terms

$$16 \mu^2 k_1 \sqrt{k_1^2 + k_2^2} + 2 a \alpha d_c^2 k_1 \sqrt{k_1^2 + k_2^2} + 4 \alpha m_\Omega (\omega - \omega_c) k_1 \sqrt{k_1^2 + k_2^2}. \quad (34)$$

The parameter α entering the superpotential is the least controlled by the fundamental symmetries and phenomenological considerations, and plays a very significant role. Small values of α make the difference between k_1 and k_2 indistinguishable in the graphs. Since the final baryon symmetry after conversion from the lepton asymmetry is a small number, such parameter ranges are also of relevance. Mid-range values of α are favorable to make the phase of $\chi = \tan^{-1}(k_2/k_1)$ more pronounced as can be seen from table III.

We see in table III that the CP phase values in both models are identical, other parameters remaining the same. This can be seen from the effective potential for MSLR \mathcal{P} worked out in the appendix B. The corresponding expression for the effective potential for MSLRM can be obtained by simply reversing the sign of ω_c . However, upon minimizing, the vev for ω_c also has opposite signs in the two models and hence the k_1, k_2 see the same effective potential in the two cases.

VIII. CONCLUSION

We have explored two possible realizations of supersymmetric Left-Right symmetric model for their implications to cosmology. The superpotential imposes the requirements that $SU(2)_R$ breaks first to $U(1)_R$ at a scale M_R and $U(1)_{B-L}$ breaks at a lower scale M_{B-L} with a see-saw requirement $M_{B-L}^2 \simeq M_R M_{EW}$ with respect to the SM scale M_{EW} . This makes it interesting to explore the values 10^4 GeV for $B-L$ breaking scale and 10^6 GeV for the $SU(2)_R$ breaking scale.

The first stage of symmetry breaking makes only local choices of the new phase, leading to formation of DW, which remain metastable down to M_{EW} temperature scale in MSLRM but only upto a higher scale m_{B-L} in MSLR \mathcal{P} . After the DW are rendered metastable, they remain a dominant source of energy down to a temperature T_D which would depend on the details of DW evolution dynamics. Only when the DW have disappeared is the phase transition completed, ensuring a unique global choice of chirality. These facts, summarized in table I play a central role in constraining the models since the DW dynamics is meant

to achieve two important cosmological goals, that of removing unwanted relics by inducing secondary or weak inflation and causing leptogenesis. Cosmologically acceptable values of T_D are shown to constrain soft parameters in the Higgs sectors of the two models in table II and eq. (17). We have presented the explicit solutions for the DW configurations for a range of parameters and determined the possibility of a transient CP violating phase in the core of the DW. It is interesting that due to the nature of the effective potential, the CP violating phase is quantitatively identical in the two variants for the same values of the parameters. This is discussed in sec. VII.

The MSLRM permits a long duration of cosmological domination by DW. The disappearance of the DW and the completion of the phase transition is signaled only after TeV scale supersymmetry breaking. This permits removal of cosmological relics, but also potentially leptogenesis from the uni-directional motion of the DW. The phase transition is expected to end with reheating to a scale above the electroweak scale, so that thermal leptogenesis mechanism through resonant leptogenesis, arising from soft supersymmetry breaking terms is also possible. It is interesting that the estimate $B \sim 0.1\text{eV}$ in the leptonic sector required from thermal leptogenesis is in concordance with the independent cosmological requirement on soft parameters in the Higgs sector for the successful disappearance of the DW.

A new model MSLR \mathcal{P} has been proposed for making global parity breakdown to a unique vacuum natural. It relies on choosing a phase -1 for the $SU(2)$ triplets Ω and Ω_c under the parity $L \leftrightarrow R$. The first order phase transition leading to unique global vacuum is signaled in this model at the higher scale M_{B-L} compared to the case of MSLRM. Successful completion of the phase transition in this model also relies on the supersymmetry breaking mechanism but it is possible to impose the stricter requirement that the soft terms obey the gauge and discrete symmetries of the superpotential. The uniqueness of the global vacuum then follows from the spontaneous symmetry breaking within the visible sector. Again, as in the MSLRM, resonant soft leptogenesis as well as DW mediated leptogenesis remain viable.

There are general arguments based on intrinsic reasons suggesting that TeV scale leptogenesis if true cannot be verified in colliders in the near future [32]. We have adopted the approach of [59] wherein cosmology requirements arising from soft resonant leptogenesis are correlated with collider observables. Furthermore, the occurrence of a phase transition accompanied by domain walls may be verifiable in upcoming and planned gravitational wave experiments [69]. An open question for this class of models is a comprehensive analysis of

the two different potential sources of leptogenesis, from phase transition DW and from the resonant thermal mechanism. Successful cumulative leptogenesis and subsequent dilution to required baryon asymmetry can further constrain the parameters of the models.

IX. ACKNOWLEDGMENT

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APPENDIX A: F AND D FLATNESS CONDITIONS

The F -flatness conditions for MSLRP are

$$\begin{aligned}
F_{\bar{\Delta}} &= m_{\Delta}\Delta + a(\Delta\Omega - \frac{1}{2}\text{Tr } \Delta\Omega) = 0 \\
F_{\bar{\Delta}_c} &= m_{\Delta}\Delta_c - a(\Delta_c\Omega_c - \frac{1}{2}\text{Tr } \Delta_c\Omega_c) = 0 \\
F_{\Delta} &= m_{\Delta}\bar{\Delta} + a(\Omega\bar{\Delta} - \frac{1}{2}\text{Tr } \Omega\bar{\Delta}) = 0 \\
F_{\Delta_c} &= m_{\Delta}\bar{\Delta}_c - a(\Omega_c\bar{\Delta}_c - \frac{1}{2}\text{Tr } \Omega_c\bar{\Delta}_c) = 0 \\
F_{\Omega} &= m_{\Omega}\Omega + a(\bar{\Delta}\Delta - \frac{1}{2}\text{Tr } \bar{\Delta}\Delta) + \alpha_{ij}\tau_2^T\Phi_j\tau_2^T\Phi_i^T = 0 \\
F_{\Omega_c} &= m_{\Omega}\Omega_c - a(\bar{\Delta}_c\Delta_c - \frac{1}{2}\text{Tr } \bar{\Delta}_c\Delta_c) - \alpha_{ij}\tau_2^T\Phi_j^T\tau_2^T\Phi_i = 0 \\
F_{\Phi_i} &= \alpha_{ij}(\Omega^T\tau_2^T\Phi_j\tau_2^T - \tau_2\Omega\Phi_j\tau_2 - \tau_2\Phi_j\tau_2\Omega_c + \tau_2^T\Phi_j\Omega_c^T\tau_2^T) \\
&\quad + \mu_{ij}(\tau_2^T\Phi_j\tau_2^T + \tau_2\Phi_j\tau_2) = 0
\end{aligned} \tag{A1}$$

The D -flatness conditions for MSLRP are given by

$$\begin{aligned}
D_{Ri} &= 2\text{Tr } \Delta_c^\dagger\tau_i\Delta_c + 2\text{Tr } \bar{\Delta}_c^\dagger\tau_i\bar{\Delta}_c + 2\text{Tr } \Omega_c^\dagger\tau_i\Omega_c = 0 \\
D_{Li} &= 2\text{Tr } \Delta^\dagger\tau_i\Delta + 2\text{Tr } \bar{\Delta}^\dagger\tau_i\bar{\Delta} + 2\text{Tr } \Omega^\dagger\tau_i\Omega = 0 \\
D_{B-L} &= 2\text{Tr } (\Delta^\dagger\Delta - \bar{\Delta}^\dagger\bar{\Delta}) - 2\text{Tr } (\Delta_c^\dagger\Delta_c - \bar{\Delta}_c^\dagger\bar{\Delta}_c) = 0
\end{aligned} \tag{A2}$$

Since the Leptons L and L_c are considered to have zero vev, we omit them from the F and D flat conditions. The above conditions are same for MSLRM with only Ω_c replaced by $-\Omega_c$.

APPENDIX B: SIMPLIFIED EFFECTIVE POTENTIAL

Here we display the simplified effective potential involving seven degrees of freedom referred to in sec. VII.

$$\begin{aligned}
V_{7dof} = & \frac{a^2}{2} \left((d_1^2 + d_2^2)^2 + d_c^4 \right) + 2a^2 (\omega^2 (d_1^2 + d_2^2) + \omega_c^2 d_c^2) \\
& + 16\mu^2 (3(k_1^2 + k_2^2) + k_1 \sqrt{k_1^2 + k_2^2}) + 16\alpha^2 (\omega - \omega_c)^2 (k_1^2 + k_2^2) \\
& + 8\alpha^2 (k_1^2 + k_2^2)^2 - 8\alpha^2 k_1 (k_1^2 + k_2^2)^{3/2} - 2a\alpha d_1 \sqrt{d_1^2 + d_2^2} (k_1^2 + k_2^2) \\
& + 2a\alpha (d_1 k_1 + d_2 k_2) \sqrt{d_1^2 + d_2^2} \sqrt{k_1^2 + k_2^2} + 2a\alpha d_c^2 k_1 \sqrt{k_1^2 + k_2^2} \\
& - 2a\alpha d_c^2 (k_1^2 + k_2^2) + 4am_\Delta (\omega(d_1^2 + d_2^2) - \omega_c d_c^2) \\
& + 2m_\Delta^2 (d_1^2 + d_2^2 + d_c^2) + 2am_\Omega \left(\omega d_1 \sqrt{d_1^2 + d_2^2} - \omega_c d_c^2 \right) \\
& - 4\alpha m_\Omega (\omega - \omega_c) \left(k_1^2 + k_2^2 - k_1 \sqrt{k_1^2 + k_2^2} \right) + 2m_\Omega^2 (\omega^2 + \omega_c^2)
\end{aligned} \tag{B1}$$

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- [1] J. C. Pati and A. Salam, Phys. Rev. **D10**, 275 (1974).
 - [2] R. N. Mohapatra and J. C. Pati, Phys. Rev. **D11**, 2558 (1975).
 - [3] G. Senjanovic and R. N. Mohapatra, Phys. Rev. **D12**, 1502 (1975).
 - [4] R. N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. **44**, 1316 (1980).
 - [5] N. G. Deshpande, J. F. Gunion, B. Kayser, and F. I. Olness, Phys. Rev. **D44**, 837 (1991).
 - [6] S. L. Glashow, Nucl. Phys. **22**, 579 (1961).
 - [7] S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).
 - [8] A. Salam, in *Elementary Particle Theory*, edited by W. Svartholm (Almquist and Wiksell, Stockholm, 1968), p. 367.
 - [9] H. Georgi, AIP Conf. Proc. **23**, 575 (1975).
 - [10] H. Fritzsch and P. Minkowski, Ann. Phys. **93**, 193 (1975).
 - [11] S. Fukuda et al. (Super-Kamiokande), Phys. Rev. Lett. **86**, 5656 (2001), hep-ex/0103033.
 - [12] Q. R. Ahmad et al. (SNO), Phys. Rev. Lett. **89**, 011301 (2002), nucl-ex/0204008.
 - [13] Q. R. Ahmad et al. (SNO), Phys. Rev. Lett. **89**, 011302 (2002), nucl-ex/0204009.
 - [14] J. N. Bahcall and C. Pena-Garay, New J. Phys. **6**, 63 (2004), hep-ph/0404061.

- [15] P. Minkowski, Phys. Lett. **B67**, 421 (1977).
- [16] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Z. Freedman (North Holland, Amsterdam, 1979), p. 315.
- [17] T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (Tsukuba, Japan, 1979), p. 95.
- [18] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
- [19] N. Sahu and U. A. Yajnik, Phys. Rev. **D71**, 023507 (2005), hep-ph/0410075.
- [20] E. Witten, Nucl. Phys. **B188**, 513 (1981).
- [21] R. K. Kaul and P. Majumdar, Nucl. Phys. **B199**, 36 (1982).
- [22] S. P. Martin, hep-ph/9709356 (1997).
- [23] I. J. R. Aitchison, hep-ph/0505105 (2005).
- [24] D. J. H. Chung et al., Phys. Rept. **407**, 1 (2005).
- [25] C. S. Aulakh, A. Melfo, and G. Senjanovic, Phys. Rev. **D57**, 4174 (1998), hep-ph/9707256.
- [26] Q. Shafi and C.-A. Lee, Phys. Lett. **B661**, 33 (2008), arXiv:0709.4637 [hep-ph].
- [27] U. A. Yajnik and A. Sarkar, AIP Conf. Proc. **903**, 685 (2007), hep-ph/0610161.
- [28] D. Chang, R. N. Mohapatra, and M. K. Parida, Phys. Rev. **D30**, 1052 (1984).
- [29] A. Sarkar and U. A. Yajnik, Phys. Rev. **D76**, 025001 (2007), hep-ph/0703142.
- [30] M. Fukugita and T. Yanagida, Phys. Lett. **B174**, 45 (1986).
- [31] S. Davidson and A. Ibarra, Phys. Lett. **B535**, 25 (2002), hep-ph/0202239.
- [32] T. Hambye, Nucl. Phys. **B633**, 171 (2002), hep-ph/0111089.
- [33] W. Buchmuller, P. Di Bari, and M. Plumacher, Nucl. Phys. **B665**, 445 (2003), hep-ph/0302092.
- [34] W. Buchmuller, P. Di Bari, and M. Plumacher, New J. Phys. **6**, 105 (2004), hep-ph/0406014.
- [35] N. Sahu, P. Bhattacharjee, and U. A. Yajnik, Phys. Rev. **D70**, 083534 (2004), hep-ph/0406054.
- [36] N. Sahu, P. Bhattacharjee, and U. A. Yajnik, Nucl. Phys. **B752**, 280 (2006), hep-ph/0512350.
- [37] R. Jeannerot and M. Postma, JCAP **0512**, 006 (2005), hep-ph/0507162.
- [38] P. Gu and H. Mao, Phys. Lett. **B619**, 226 (2005), hep-ph/0503126.
- [39] A. Stern and U. A. Yajnik, Nucl. Phys. **B267**, 158 (1986).
- [40] M. Kawasaki and K. Maeda, Phys. Lett. **B208**, 84 (1988).
- [41] A. Davis and M. A. Earnshaw, Nucl. Phys. **B394**, 21 (1993).
- [42] R. Jeannerot, Phys. Rev. Lett. **77**, 3292 (1996), hep-ph/9609442.

- [43] W. Fischler, G. F. Giudice, R. G. Leigh, and S. Paban, Phys. Lett. **B258**, 45 (1991).
- [44] Y. Grossman, T. Kashti, Y. Nir, and E. Roulet, Phys. Rev. Lett. **91**, 251801 (2003), hep-ph/0307081.
- [45] G. D'Ambrosio, G. F. Giudice, and M. Raidal, Phys. Lett. **B575**, 75 (2003), hep-ph/0308031.
- [46] L. Boubekur, T. Hambye, and G. Senjanovic, Phys. Rev. Lett. **93**, 111601 (2004), hep-ph/0404038.
- [47] E. J. Chun and S. Scopel, Phys. Lett. **B636**, 278 (2006), hep-ph/0510170.
- [48] A. Pilaftsis, Phys. Rev. **D56**, 5431 (1997), hep-ph/9707235.
- [49] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, Ann. Rev. Nucl. Part. Sci. **43**, 27 (1993), hep-ph/9302210.
- [50] J. M. Cline, U. A. Yajnik, S. N. Nayak, and M. Rabikumar, Phys. Rev. **D66**, 065001 (2002), hep-ph/0204319.
- [51] R. Kuchimanchi and R. N. Mohapatra, Phys. Rev. **D48**, 4352 (1993), hep-ph/9306290.
- [52] R. Kuchimanchi and R. N. Mohapatra, Phys. Rev. Lett. **75**, 3989 (1995), hep-ph/9509256.
- [53] M. Cvetič, Phys. Lett. **B164**, 55 (1985).
- [54] C. S. Aulakh, K. Benakli, and G. Senjanovic, Phys. Rev. Lett. **79**, 2188 (1997), hep-ph/9703434.
- [55] C. S. Aulakh, A. Melfo, A. Rasin, and G. Senjanovic, Phys. Rev. **D58**, 115007 (1998), hep-ph/9712551.
- [56] J. Preskill, S. P. Trivedi, F. Wilczek, and M. B. Wise, Nucl. Phys. **B363**, 207 (1991).
- [57] M. Kawasaki and F. Takahashi, Phys. Lett. **B618**, 1 (2005), hep-ph/0410158.
- [58] M. Dine and A. E. Nelson, Phys. Rev. **D48**, 1277 (1993), hep-ph/9303230.
- [59] E. J. Chun, Phys. Rev. **D72**, 095010 (2005), hep-ph/0508050.
- [60] E. J. Chun, Phys. Rev. **D69**, 117303 (2004), hep-ph/0404029.
- [61] A. E. Nelson, D. B. Kaplan, and A. G. Cohen, Nucl. Phys. **B373**, 453 (1992).
- [62] J. M. Frere, L. Houart, J. M. Moreno, J. Orloff, and M. Tytgat, Phys. Lett. **B314**, 289 (1993), hep-ph/9301228.
- [63] M. Joyce, T. Prokopec, and N. Turok, Phys. Rev. Lett. **75**, 1695 (1995), hep-ph/9408339, Erratum-ibid. 75:3375, (1995).
- [64] M. Joyce, T. Prokopec, and N. Turok, Phys. Rev. **D53**, 2930 (1996), hep-ph/9410281.
- [65] J. M. Cline, M. Joyce, and K. Kainulainen, Phys. Lett. **B417**, 79 (1998), hep-ph/9708393,

Erratum-ibid.B448:321, (1999).

- [66] J. M. Cline, M. Joyce, and K. Kainulainen, JHEP **07**, 018 (2000), hep-ph/0006119.
- [67] J. M. Cline and K. Kainulainen, Phys. Rev. Lett. **85**, 5519 (2000), hep-ph/0002272.
- [68] M.-C. Chen and K. T. Mahanthappa, Phys. Rev. **D71**, 035001 (2005), hep-ph/0411158.
- [69] C. Grojean and G. Servant, Phys. Rev. **D75**, 043507 (2007), hep-ph/0607107.